

NAG Toolbox for MATLAB

f04bh

1 Purpose

f04bh computes the solution to a real system of linear equations $AX = B$, where A is an n by n symmetric matrix and X and B are n by r matrices. An estimate of the condition number of A and an error bound for the computed solution are also returned.

2 Syntax

```
[a, ipiv, b, rcond, errbnd, ifail] = f04bh(uplo, a, b, 'n', n, 'nrhs_p',
nrhs_p)
```

3 Description

The diagonal pivoting method is used to factor A as $A = UDU^T$, if **uplo** = 'U', or $A = LDL^T$, if **uplo** = 'L', where U (or L) is a product of permutation and unit upper (lower) triangular matrices, and D is symmetric and block diagonal with 1 by 1 and 2 by 2 diagonal blocks. The factored form of A is then used to solve the system of equations $AX = B$.

4 References

Anderson E, Bai Z, Bischof C, Blackford S, Demmel J, Dongarra J J, Du Croz J J, Greenbaum A, Hammarling S, McKenney A and Sorensen D 1999 *LAPACK Users' Guide* (3rd Edition) SIAM, Philadelphia URL: <http://www.netlib.org/lapack/lug>

Higham N J 2002 *Accuracy and Stability of Numerical Algorithms* (2nd Edition) SIAM, Philadelphia

5 Parameters

5.1 Compulsory Input Parameters

1: **uplo** – string

If **uplo** = 'U', the upper triangle of the matrix A is stored.

If **uplo** = 'L', the lower triangle of the matrix A is stored.

Constraint: **uplo** = 'U' or 'L'.

2: **a(lda,*)** – double array

The first dimension of the array **a** must be at least $\max(1, \mathbf{n})$

The second dimension of the array must be at least $\max(1, \mathbf{n})$

The n by n symmetric matrix A .

If **uplo** = 'U', the leading $\mathbf{n}$ by $\mathbf{n}$ upper triangular part of the array **a** contains the upper triangular part of the matrix A , and the strictly lower triangular part of **a** is not referenced.

If **uplo** = 'L', the leading $\mathbf{n}$ by $\mathbf{n}$ lower triangular part of the array **a** contains the lower triangular part of the matrix A , and the strictly upper triangular part of **a** is not referenced.

3: **b(lb,*)** – double array

The first dimension of the array **b** must be at least $\max(1, \mathbf{n})$

The second dimension of the array must be at least $\max(1, \mathbf{nrhs_p})$. To solve the equations $Ax = b$, where b is a single right-hand side, $\mathbf{b}$ may be supplied as a one-dimensional array with length $\mathbf{ldb} = \max(1, \mathbf{n})$

The n by r matrix of right-hand sides B .

5.2 Optional Input Parameters

1: $\mathbf{n}$ – int32 scalar

Default: The second dimension of the array $\mathbf{a}$.

The number of linear equations n , i.e., the order of the matrix A .

Constraint: $\mathbf{n} \geq 0$.

2: $\mathbf{nrhs_p}$ – int32 scalar

Default: The second dimension of the array $\mathbf{b}$.

The number of right-hand sides r , i.e., the number of columns of the matrix B .

Constraint: $\mathbf{nrhs_p} \geq 0$.

5.3 Input Parameters Omitted from the MATLAB Interface

$\mathbf{lda}, \mathbf{ldb}$

5.4 Output Parameters

1: $\mathbf{a}(\mathbf{lda},*)$ – double array

The first dimension of the array $\mathbf{a}$ must be at least $\max(1, \mathbf{n})$

The second dimension of the array must be at least $\max(1, \mathbf{n})$

If $\mathbf{ifail} \geq 0$, the block diagonal matrix D and the multipliers used to obtain the factor U or L from the factorization $A = UDU^T$ or $A = LDL^T$ as computed by f07md.

2: $\mathbf{ipiv}(*)$ – int32 array

Note: the dimension of the array $\mathbf{ipiv}$ must be at least $\max(1, \mathbf{n})$.

If $\mathbf{ifail} \geq 0$, details of the interchanges and the block structure of D , as determined by f07md.

$\mathbf{ipiv}(k) > 0$

Rows and columns k and $\mathbf{ipiv}(k)$ were interchanged, and d_{kk} is a 1 by 1 diagonal block.

$\mathbf{uplo} = 'U'$ and $\mathbf{ipiv}(k) = \mathbf{ipiv}(k - 1) < 0$

Rows and columns $k - 1$ and $-\mathbf{ipiv}(k)$ were interchanged and $d_{k-1:k,k-1:k}$ is a 2 by 2 diagonal block.

$\mathbf{uplo} = 'L'$ and $\mathbf{ipiv}(k) = \mathbf{ipiv}(k + 1) < 0$

Rows and columns $k + 1$ and $-\mathbf{ipiv}(k)$ were interchanged and $d_{k:k+1,k:k+1}$ is a 2 by 2 diagonal block.

3: $\mathbf{b}(\mathbf{ldb},*)$ – double array

The first dimension of the array $\mathbf{b}$ must be at least $\max(1, \mathbf{n})$

The second dimension of the array must be at least $\max(1, \mathbf{nrhs_p})$. To solve the equations $Ax = b$, where b is a single right-hand side, $\mathbf{b}$ may be supplied as a one-dimensional array with length $\mathbf{ldb} = \max(1, \mathbf{n})$

If $\mathbf{ifail} = 0$ or $Np1$, the n by r solution matrix X .

4: **rcond** – double scalar

If **ifail** ≥ 0 , an estimate of the reciprocal of the condition number of the matrix A , computed as $\mathbf{rcond} = 1/(\|A\|_1 \|A^{-1}\|_1)$.

5: **errbnd** – double scalar

If **ifail** = 0 or $Np1$, an estimate of the forward error bound for a computed solution $\hat{x}$, such that $\|\hat{x} - x\|_1 / \|x\|_1 \leq \mathbf{errbnd}$, where $\hat{x}$ is a column of the computed solution returned in the array **b** and x is the corresponding column of the exact solution X . If **rcond** is less than *machine precision*, then **errbnd** is returned as unity.

6: **ifail** – int32 scalar

0 unless the function detects an error (see Section 6).

6 Error Indicators and Warnings

Errors or warnings detected by the function:

ifail < 0 and **ifail** $\neq -999$

If **ifail** = $-i$, the i th argument had an illegal value.

ifail = -999

Allocation of memory failed. The integer allocatable memory required is **n**, and the double allocatable memory required is $\max(2 \times \mathbf{n}, \mathbf{lwork})$, where **lwork** is the optimum workspace required by f07ma. If this failure occurs it may be possible to solve the equations by calling the packed storage version of f04bh, f04bj, or by calling f07ma directly with less than the optimum workspace (see Chapter F07).

ifail > 0 and **ifail** $\leq N$

If **ifail** = i , d_{ii} is exactly zero. The factorization has been completed, but the block diagonal matrix D is exactly singular, so the solution could not be computed.

ifail = $N + 1$

rcond is less than *machine precision*, so that the matrix A is numerically singular. A solution to the equations $AX = B$ has nevertheless been computed.

7 Accuracy

The computed solution for a single right-hand side, $\hat{x}$, satisfies an equation of the form

$$(A + E)\hat{x} = b,$$

where

$$\|E\|_1 = O(\epsilon)\|A\|_1$$

and ϵ is the *machine precision*. An approximate error bound for the computed solution is given by

$$\frac{\|\hat{x} - x\|_1}{\|x\|_1} \leq \kappa(A) \frac{\|E\|_1}{\|A\|_1},$$

where $\kappa(A) = \|A^{-1}\|_1 \|A\|_1$, the condition number of A with respect to the solution of the linear equations. f04bh uses the approximation $\|E\|_1 = \epsilon \|A\|_1$ to estimate **errbnd**. See Section 4.4 of Anderson *et al.* 1999 for further details.

8 Further Comments

The total number of floating-point operations required to solve the equations $AX = B$ is proportional to $(\frac{1}{3}n^3 + 2n^2r)$. The condition number estimation typically requires between four and five solves and never more than eleven solves, following the factorization.

In practice the condition number estimator is very reliable, but it can underestimate the true condition number; see Section 15.3 of Higham 2002 for further details.

The complex analogues of f04bh are f04ch for complex Hermitian matrices, and f04dh for complex symmetric matrices.

9 Example

```

uplo = 'Upper';
a = [-1.81, 2.06, 0.63, -1.15;
      0, 1.15, 1.87, 4.2;
      0, 0, -0.21, 3.87;
      0, 0, 0, 2.07];
b = [0.96, 3.93;
      6.07, 19.25;
      8.38, 9.9;
      9.5, 27.85];
[aOut, ipiv, bOut, rcond, errbnd, ifail] = f04bh(uplo, a, b)

aOut =
    0.4074    0.3031   -0.5960    0.6537
         0   -2.5907    0.8115    0.2230
         0         0    1.1500    4.2000
         0         0         0    2.0700

ipiv =
         1
         2
        -2
        -2

bOut =
   -5.0000    2.0000
   -2.0000    3.0000
    1.0000    4.0000
    4.0000    1.0000

rcond =
    0.0132

errbnd =
   8.4111e-15

ifail =
         0

```